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Slope Angle and Slope Length Solutions for GIS

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ABSTRACT

The Universal Soil Loss Equation has been used for a number of years to estimate soil erosion. One of its parameters is slope length, however, slope length has traditionally been estimated for large areas rather than calculated. Using data from regular grid DEMs, a method is described in this paper for calculating the cumulative downhill slope length. In addition, methods for calculating slope angle and downhill direction (aspect) are defined. Details of the algorithm and its associated advantages and disadvantages are discussed.

INTRODUCTION

Slope angle and slope length calculations are an integral part of many environmental analyses, particularly erosion models. Unfortunately, there are problems with most of the methods currently available for the calculation of these parameters.

Typical slope angle computation methods calculate an average slope based upon, roughly, a 3x3 neighbourhood (Fairfield and Leymarie 1991). The maximum slope method calculates the maximum angle to or from the centre cell – the result of this being much higher overall slope angle estimates (and resulting erosion estimates). The proposed solution to these

problems (averaging or overestimating) is to calculate maximum downhill slope angle – constraining the slope angle calculations to one cell length (or 1.4 cell lengths in the diagonal) in a downhill direction. This slope angle also corresponds to the direction of overland flow from the cell in question. For more information regarding different slope angle calculations, see Dunn and Hickey (1998) and Srinivasan and Engel (1991).

Slope length calculations are often the most problematic of the erosion model parameters. The traditional method of calculation has been to use a regional estimate – thereby converting a variable into a constant (Troeh, *et al.*, 1991). Most recent work has centred around using either unit stream power (Moore and Burch, 1986) or upslope area (Desmet and Govers, 1996) as a surrogate for slope length. The methodology described in this paper involves calculating slope length from high points (ridges/peaks) along the direction of maximum downhill slope angle (flowdirection). Converging flows and areas of deposition are accounted for in the algorithm.

EROSION MODELLING USING THE USLE

The Universal Soil Loss Equation (USLE) has been used for a number of years to predict soil erosion rates. In its traditional form, the USLE is given by the following equation (Wischmeier and Smith, 1978):

$$A = R K L S C P$$

Where A is the average annual soil loss per unit area

R is the rainfall and runoff factor

K is the soil erodability factor

L is the slope length factor

S is the slope steepness factor

C is the cover and management factor

P is the support practice factor

The L and S factors are commonly combined as LS and referred to as the Slope factor (Troeh, *et al.*, 1991). This paper will focus on generating the L and S factors from a digital elevation model (DEM) within a GIS. The two inputs to the LS factor are cumulative slope length and slope angle.

In recent years, soil erosion models more advanced than the USLE have been developed, including ANSWERS (Beasley and Huggins, 1991), AGNPS (Young *et al.*, 1987), WEPP (Laflen *et al.*, 1991; Savabi, *et al.*, 1995), and the USLE's replacement, RUSLE (Renard *et al.*, 1997; Renard *et al.*, 1991). It is important to note that all models include a slope component and that the RUSLE, WEPP, and AGNPS models include a slope length component in their equations.

METHODS OF SLOPE ANGLE CALCULATION

Calculating slope from a DEM is relatively simple, but care must be taken when selecting an algorithm. For example, the ARC/INFO GRID command, **SLOPE**, uses the quadratic surface algorithm by Srinivasan and Engel (1991) (ESRI, 1997); IDRISI's algorithm calculates maximum slope, but only considers the four cardinal directions (N S E W) from a 3x3 neighbourhood (IDRISI, 1997). Other available algorithms include the neighbourhood method and the best fit plane method (Srinivasan and Engel, 1991). These, along with the quadratic surface method, examine the neighbourhood around an elevation cell and determine the slope across the cell in question based upon at least four of the neighbour cells. As such, all three calculate an average slope based upon, roughly, a 3x3 neighbourhood. The maximum slope method calculates the maximum angle to or from the centre cell – the result of this being much higher overall slope angle estimates (and resulting erosion estimates) (Srinivasan and Engel, 1991; Srinivasan, *et al.*, 1994; Dunn and Hickey, 1997).

Given the limitations in the above methods (averages across a 3x3 neighbourhood, maximum values, and only using cardinal directions), a compromise method was required to retain local variability without focusing entirely on maximum slope angles.

This method is the maximum downhill slope angle which constrains the slope angle calculations to one cell length (or 1.4 cell lengths in the diagonal) in a downhill direction. It is similar to the maximum slope method, but it includes a directional component -- angles are constrained to a downhill direction (uphill angles are calculated as having a negative slope). The obvious limitation with this method is negative slope angles are produced when dealing with depressions in a DEM. A side benefit is that the slope angle also corresponds to the direction of overland flow from the cell in question (Dunn and Hickey, 1997).

Computer code to calculate maximum downhill slope angle (and cumulative downhill

slope length) are available for both Arc Info (AML format) and IDRISI (executable code). They can be downloaded from <http://www.cage.curtin.edu.au/~rhickey/slope.html> and are free.

METHODS OF SLOPE LENGTH CALCULATION

Generating the LS values poses the largest problem in using the USLE (Griffin *et al.*, 1988; Moore and Wilson, 1992; Renard, *et al.*, 1991), especially when applying it to real landscapes within a GIS (Griffin *et al.*, 1988). Traditionally, the best estimates for L are obtained from field measurements, but these are rarely available or practical. Unfortunately, because of the lack of detailed slope length measurements or reliable software algorithms, regional average slope length values are often used (Cowen, 1993) – thereby treating a variable as a constant.

The particular algorithms that have been developed to calculate slope length include grid-based methods (Hickey *et al.*, 1994; Hickey, 1994), unit stream power theory (Mitasova, 1993; Mitasova, *et al.*, 1996, Moore and Wilson, 1992; Moore and Burch, 1986), contributing area (Desmet and Govers, 1995; Desmet and Govers, 1996), and Cowen's (1993) study which developed the means to calculate cumulative downhill slope length from a TIN (triangular irregular network) within ARC/INFO.

GRID-BASED ALGORITHM DESCRIPTION:

The overall methodology for calculating the L and S factors is illustrated in Figure 1. The first requirement for the algorithm is a DEM, preferably a depressionless DEM. This is suggested for two reasons. First, true depressions are rare in nature, as such, depressions in DEMs are often errors. Second, when using the suggested maximum downhill slope angle algorithm, depressions will return negative slope values. This will eventually result in negative erosion estimates (deposition). AML code is available for ARC/INFO which eliminates all depressions (the FILL command in ARC/INFO GRID does not eliminate all depressions), unfortunately, no similar code is available for IDRISI.

Once this has been completed, the maximum downhill slope and the flowdirection are calculated from the DEM. It is important to note here that the flowdirection and the direction of maximum downhill slope are the same.

High points (local maxima) are designated by selecting those cells which have either no

flow entering them or in cases where both the cell in question and its input cell have a slope angle of zero. For accurate cumulative slope length values, the high points calculated by the model must correspond to true terrain high points. From a geomorphic perspective, these primarily include ridges and peaks, however, there are often other parts of the landscape that do not receive flow from upslope. To accomplish the identification of high points, the model must be run on hydrologically isolated areas (ie. watershed scale, not on partial watersheds). If this is not done, slope lengths are underestimated because the high points calculated may be located on the sides of slopes, not on ridgelines.

Non-cumulative slope length (NCSL) is then calculated for each cell within the watershed. In short, the distance to each cell from its input cell is calculated using the following equations:

if the cell being calculated is a high point
 then NCSL = 0.5(cell resolution)
 if the input cell is in a cardinal direction (N, S, E, W),
 then NCSL = (cell resolution)
 otherwise,
 NCSL = 1.4142(cell resolution)

This calculation is based on the assumption that the calculations for slope length are from the centre of the cell to the center of its input cell. Therefore, as high points do not have an input cell, the 0.5 value represents only the erosion occurring within the half of that cell that is uphill of the centre point. The measurements are done within (x,y) space rather than (x,y,z) space to conform to USLE and RUSLE input requirements (Renard, *et al.*, 1997).

At this point, all the required input grids are present in order to calculate the cumulative slope length (ie NCSL, slope angle, high points and flowdirection). Curriculum slope length is computed by simply summing the non-cumulative slope lengths along the flow direction beginning at the high points.

There are a number of assumptions built into this calculation. The first is that in areas of converging flows, the highest cumulative slope length takes precedence. The second assumption to be considered is for areas where deposition, not erosion, is the dominant process. As such, the

code includes a mechanism for defining areas of deposition (useful when calculating erosion), called the *cutoff slope angle*. When considering water flowing across the earth's surface carrying sediment, the flow velocity will decrease enough at some points so that the sediment carried will be deposited – rather than the energy of the flow enabling more sediment (erosion) to be picked up. The critical factor is to define those parts of the landscape where this will happen. The answer is primarily a function of two things: the slope of the land (which very much defines the velocity), and the saturation of the flow. If the flow is fully saturated (with sediment), any decrease in velocity will result in deposition rather than erosion. On the other hand, if the flow is relatively unsaturated, it will take a very significant decrease in slope (possibly to zero) to result in deposition.

This particular problem is addressed by the *cutoff slope angle* which is a user-input value. The *cutoff slope angle* is defined as the change in slope angle from one cell to the next along the *flowdirection*. This value ranges from 0 to 1 for all areas where the slope angle decreases from one cell to the next (if the slope angle increases, there will definitely be no deposition). Therefore, the user input value will range from 0 – 1 and be dependent upon the amount of sediment carried by overland flow. For example, an input value of zero will cause the slope length to reset every time there is a decrease in slope angle. An input value of one will cause the slope length to never reset. In an ideal world, this value would be set by an expert familiar with the particular area in question. However, this is not always feasible. The literature (Griffin, *et al.*, 1988; Wilson, 1986) suggests references that a value closer to 0.5 (slope decreasing by 50% or greater) is appropriate. This is similar to assumptions made in other studies (Griffin, *et al.*, 1988; Wilson, 1986), although in these cases, the cutoffs were a change of 50% from the average uphill slope angle and a change of 50% from the maximum uphill slope angle, respectively. It is important to note that the algorithms described in this paper only consider the nearest upslope cell in the cutoff calculations – not an average upslope or maximum uphill slope angle. However, the program does allow the user to specify any cutoff value.

The final steps in this analysis are to convert the cumulative slope length values into feet (if necessary) and to calculate the LS values. The USLE equation for LS values is:

$$LS = (\ell/72.6)^m (65.41 \sin^2\beta + 4.56\sin\beta + 0.065)$$

where ℓ is the cumulative slope length in feet;

β is the downhill slope angle;

m is a slope contingent variable;

0.5 if the slope angle is greater than 2.86° ; 0.4 on slopes of 1.72° to 2.86° ;

0.3 on slopes of 0.57° to 1.72° ;

0.2 on slopes less than 0.57° (Wischmeier and Smith, 1978).

The ARC/INFO AML program that calculates the LS values requires a DEM coverage and two input values: the cell resolution units (feet or metres) and *the cutoff slope angle* required for identifying cells with net deposition rather than erosion (a 50% decrease is the default). The IDRISI executable is run from a DOS window and requires the input of the DEM filename, output filenames, and the *cutoff slope angle* (no default given). The final outputs from the AML are three coverages: *ls_values* (USLE LS values), *slope_angle* (downhill slope angles), and *slope_len* (the cumulative slope lengths). The IDRISI code outputs the following: maximum downhill slope angle, non-cumulative slope length, cumulative downhill slope length, and the flowdirection. In this case, all filenames are input by the user.

Figure 2 illustrates the results of the calculation on a 5x5 test grid. The DEM (Figure 2a) is relatively small and has a 100 metres resolution – designed for easy understanding of the results of the code. The cumulative downhill slope length is shown in Figure 2e using a cutoff slope of 0.5. For comparison, Figure 2e also shows the result of using different slope cutoff values of 0.25. At a cutoff slope angle of 0.5, two slope lengths are reset to zero; when the cutoff slope angle is set to 0.25, the changes in slope are not enough to reset the cumulative slope lengths to zero.

DEM PROBLEMS AND LIMITATIONS:

It is important to note that there are a number of problems unique to DEMs that need to be addressed, as all may impact upon slope angle and slope length calculations.

- The first involves the many depressions (or pits) that are common on DEMs (Quinn *et al.* 1991). Real or not, in all cases they will interrupt the flow of water downhill (according to the GIS). The slope length algorithm recognises these cells as areas of deposition and resets the slope length to zero.
- The second problem is associated with striping; systematic errors that give parts of

the map a *boxy* appearance. These are errors in the DEM that are often a result of the DEM creation process and will cause errors in any DEM-based analysis.

- Third is the typically low resolution of DEMs (ie. 30m for 7.5 minute USGS DEMs). Microfeatures which slow (or increase) runoff, and therefore erosion, are lost. Thus, erosion estimates will be in error. As DEM resolution and accuracy increase, the landscape will be more accurately described and erosion estimates will approach actual values.
- Finally, there are often problems when joining two separate DEMs. For example, joining two DEMs may result in a apparent cliff running across the map.

In many cases, these problems cannot be averted without re-creating the DEM – either via photogrammetric methods or digitizing existing large scale contour maps. Therefore, DEM users must be aware of potential errors in their datasets and consider the results of these errors in their final products (maps, reports, etc.). See Fahsi, *et al.* (1990) for more details concerning the formation of and problems associated with DEMs.

MODEL ADVANTAGES

The maximum downhill slope angle calculations have the following advantages:

- By considering slope angle as a function of only two cells, the local variability is retained (no averaging across 3 cells).
- The maximising effects of the maximum slope angle calculation method are reduced by constraining slope angles to the downhill direction.

To avoid using regional averages for slope length calculations, cumulative slope length calculations are the only practical alternative if erosion rates are to be modelled. The advantages of the grid-based model are:

- The algorithm considers both areas of deposition and converging flows when calculating cumulative downhill slope length. When passed through the LS calculations, those areas in which cumulative slope length is set to zero will have a zero value for LS -- which results in a zero value for erosion.
- The cumulative slope length output can be used in a number of different erosion models, including the USLE, RUSLE, and AGNPS (Wischmeier and

Smith, 1978; Renard, *et al.*, 1991; Young, *et al.*, 1987).

- The erosion rates coverage can be used within the GIS as an input to land suitability analysis problems. For example, erosion rates may be one factor considered when deciding which parts of a large, environmentally damaged site should be reclaimed (Hickey, 1994; Hickey, *et al.*, 1997).
- Erosion rates can be calculated for large areas without time-consuming and costly slope length field surveys.

SUMMARY

Due to limitations in memory, calculation speed and availability of data, hydrologic modelling is limited to using DEMs with relatively coarse resolutions. Most microfeatures are lost with cell lengths greater than 5 metres and most models attempt to average data across three neighbouring cells which is 90 metres on a standard one second DEM (Haddock, 1996). Also, other models may calculate the slope length using a maximum difference in a 3x3 neighbourhood with the effect that this approach tends to exaggerate the slope lengths of the individual cells.

The model described in this paper provides an alternative to some of these shortcomings by calculating the cumulative uphill length from each cell which also accounts for convergent flow paths and depositional areas. Using the Slope Length model, more accurate slope length predictions can be assessed for use in the Universal Soil Loss Equation and other hydrologic models.

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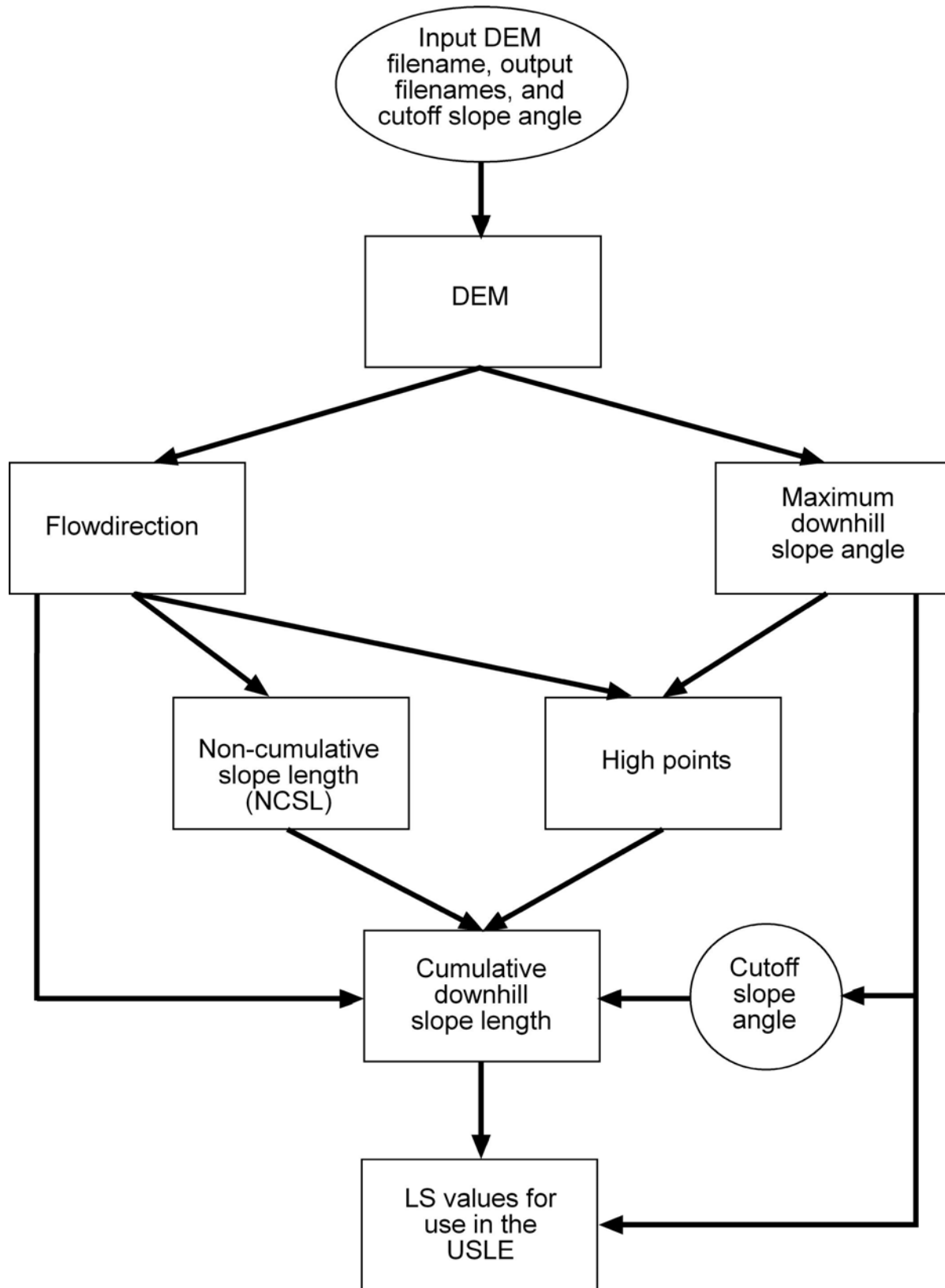


Figure 1. Flowchart illustrating the process of calculating cumulative downhill slope length (and LS values) for use in the USLE.

150	125	125	135	150
125	115	175	130	135
120	110	100	115	120
115	100	90	100	130
105	95	80	90	120

Figure 2a

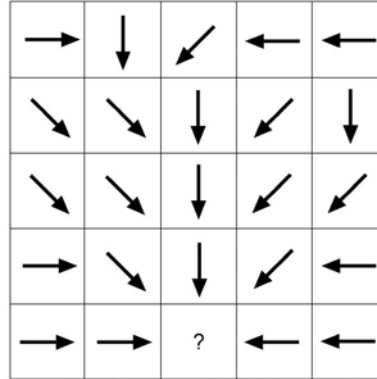


Figure 2b

14.04	5.71	4.04	5.71	8.53
6.05	6.05	36.87	11.98	8.53
8.05	8.05	5.71	10.02	8.05
8.53	8.05	5.71	8.05	16.70
5.71	8.53	0	5.71	16.70

Figure 2c

50	100	141	100	50
71	141	50	71	50
71	141	100	71	141
50	141	100	141	50
50	100	0	100	50

Figure 2d

50	0 (*150)	291	150	50
71	432	50	71	50
71	212	532	71	191
50	212	632	332	50
50	150	0	0 (*150)	50

Figure 2e

Figure 2. Example code output from test DEM

Figure 2a is the test DEM, cell resolution is 100 meters.

Figure 2b is flowdirection (aspect), measured from the cell in question downhill along the maximum downhill slope angle.

Figure 2c is the maximum downhill slope angle.

Figure 2d is non-cumulative downhill slope length (in metres) from the centre of cell to the centre of the next cell along flowdirection. To conform to USLE/RUSLE input requirements, this measurement is in x,y space, not x, y, z.

Figure 2e is cumulative downhill slope length (in metres) with a cutoff slope = 0.5.

For comparison purposes, the code was re-run using a cutoff slope of 0.25. Where the values were different than those calculated with a cutoff of 0.5, they are shown in parentheses with a "*".